

A High Efficiency Hardware Design for the Post-Quantum KEM HQC

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² The quantum threat

- Today we assist to continuous advancements in the computational capabilities of quantum computers: >1000 of qubits in 2023
- Shor's algorithm speeds up part of the cryptanalysis of **all** currently deployed asymmetric algorithms
- In 2016 NIST started the Post-Quantum standardization process
	- CRYSTALS-Kyber (FIPS 203), CRYSTALS-Dilithium (FIPS 204), SPHINCS+ (FIPS 205), FALCON (WIP)
	- Portfolio variety: standardize also a code-based scheme among Classic McEliece, BIKE, **HQC**, and a new call for digital signatures

We focused on HQC due to its strong security properties, providing a full RTL hardware accelerator:

- having a flexible architecture for the binary polynomial arithmetics
- proposing new approach for the modulo operation during the sample of polynomials: no use of DSP while providing low latency
- using the state-of-the-art algebraic encoders and decoders and adapting them for the HQC public error correction code
- suggesting an optimization to the HQC algorithm improving the overall performance of the scheme

Algebraic structure: binary polynomial ring

■ **R**: polynomial ring $\mathbb{F}_2[X]/\langle X^p - 1 \rangle$, where p is a prime number

a=*a*₀ + *a*₁*x* + . . . + *a*_{*p*−1}*x*^{*p*−1} ∈ **R** stored as vector **a**=[*a*₀, *a*₁, , *a*_{*p*−1}]

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Moderate Density Parity Check code \implies *w* ≈ \sqrt{p} Since $a_i \in \mathbb{F}_2$, an element $a \in \mathbb{R}_w$ stored as vector of the non-zero *i*

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polynomial addition (+**)**

Coefficient-wise addition: XOR (⊕) boolean operator as coefficients in \mathbb{F}_2

$$
a = a_0 + a_1x + \ldots + a_{p-1}x^{p-1}
$$

\n
$$
b = b_0 + b_1x + \ldots + b_{p-1}x^{p-1}
$$

\n
$$
a + b = (a_0 \oplus b_0) + (a_1 \oplus b_1)x + \ldots + (a_{p-1} \oplus b_{p-1})x^{p-1}
$$

$$
[a_0\oplus b_0, a_1\oplus b_1, \ldots, a_{p-1}\oplus b_{p-1}]
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polynomial subtraction (−**)**

Coefficient-wise subtraction: XOR (⊕) boolean operator as coefficients in \mathbb{F}_2

$$
\begin{array}{rcl}\na &=& a_0 &+& a_1 x &+ \ldots + & a_{p-1} x^{p-1} \\
b &=& b_0 &+& b_1 x &+ \ldots + & b_{p-1} x^{p-1} \\
\hline a - b &=& (a_0 \oplus b_0) + (a_1 \oplus b_1) x + \ldots + (a_{p-1} \oplus b_{p-1}) x^{p-1} \\
&[a_0 \oplus b_0, a_1 \oplus b_1, \ldots, a_{p-1} \oplus b_{p-1}]\n\end{array}
$$

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polynomial multiplication (·**)**

 C yclic convolution: $c_i = \bigoplus_{j+k \equiv i \bmod p} (a_j \otimes b_k), i, j, k \in \{0, 1, \ldots, p-1\}$

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a = a_0 + a_1x + \ldots + a_{p-1}x^{p-1}
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acc = (a_0 \oplus b_0) + (a_0 \oplus b_1)x + \ldots + (a_0 \oplus b_{p-1})x^{p-1}
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(a_1 \oplus b_0)x + ... + (a_1 \oplus b_{p-2})x^{p-1} + (a_1 \oplus b_{p-1})x^p
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$$

 $(a_{p-1}\oplus b_1) + (a_{p-1}\oplus b_2)x + \ldots + (a_{p-1}\oplus b_0)x^{p-1}$

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. .

if $a \in \mathbf{R}_w$ and $b \in \mathbf{R}$, has asymptotic complexity $\Theta(pw) = \Theta(p\sqrt{p}) = \Theta(p^{1.5})$

Error correction code

■ quasi-cyclic random [2*p*, *p*, *d*] code with a public parity-check matrix $H = [I_p | rot(h)]$

$$
\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 & h_0 & h_{p-1} & h_{p-2} & \cdots & h_1 \\ 0 & 1 & 0 & \cdots & 0 & h_1 & h_0 & h_{p-1} & \cdots & h_2 \\ 0 & 0 & 1 & \cdots & 0 & h_2 & h_1 & h_0 & \cdots & h_3 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & h_{p-1} & h_{p-2} & h_{p-3} & \cdots & h_0 \end{bmatrix}
$$

h is a random vector generated from the public key seed

- quasi-cyclic random [2*p*, *p*, *d*] code with a public parity-check matrix $H = [I_n | \text{rot}(h)]$
- **■** public $[n_e n_i, k_e k_i, d_e d_i]$ fixed code generated by a shortened Reed-Solomon (RS) [*ne*, *ke*, *de*] (external) code with a duplicated Reed-Muller (RM) $[n_i, k_i, d_i]$ (internal) code such that $n_e n_i \approx p$.

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NIST provided a security level classification to match the security margin of AES symmetric key encryption algorithm:

Table: Security level classification by NIST

Each HQC parameter set specifies a different algebraic structure and public error correction code.

In case both operands are in **R**:

operand1

operand2

result

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In case both operands are in **R**:

a access data in blocks of $B = 128$ bits

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In case operand1 in \mathbf{R}_w and operand2 is in \mathbf{R} : For each index *i* in the vector of operand1:

operand1

544 284 302 1402 239 819 265 1053

0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |10 | 11 operand2/result

In case operand1 in \mathbf{R}_w and operand2 is in \mathbf{R} : For each index *i* in the vector of operand1:

 \blacksquare determine the operand2 block index as $\lfloor i/B \rfloor$

operand1

operand2/result

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- \blacksquare determine the operand2 block index as $\lfloor i/B \rfloor$
- \blacksquare flip a single bit of that block by generating $T = 1 \ll (i \mod B)$

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- \blacksquare determine the operand2 block index as $\lfloor i/B \rfloor$
- \blacksquare flip a single bit of that block by generating $T = 1 \ll (i \mod B)$
- cannot be easily pipelined due to read-after-write dependency!

operand1

544 284 302 1402 239 819 265 1053

Polynomial multiplier: $\mathbf{R} \times \mathbf{R}_w$ and \mathbf{R}

- One operand is always in **R**_{*w*}
- The low weight of polynomial ($\approx \sqrt{p}$) makes the schoolbook shift-and-add approach interesting: $\varTheta(\boldsymbol{\rho}^{1.5})$ asymptotic complexity

Polynomial multiplier: $\mathbf{R} \times \mathbf{R}_w$ and \mathbf{R}

- One operand is always in **R**_{*w*}
- The low weight of polynomial ($\approx \sqrt{p}$) makes the schoolbook shift-and-add approach interesting: $\varTheta(\boldsymbol{\rho}^{1.5})$ asymptotic complexity
- There are faster algorithms based on the NTT with better asymptotic complexity, but:
	- the polynomial ring is not compatible with any NTT algorithm
	- memory access pattern is challenging to optimize

Single index processed

start block = $\left| \frac{(\rho - i)}{B} \right|$ shift amount = $\lfloor (p - i) \bmod B \rfloor$

operand1

1332 862 302 1402 239 819 265 1053

accumulator

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Single index processed

start block = $\left| \frac{(\rho - i)}{B} \right|$ shift amount = $| (p - i) \text{ mod } B |$

operand1

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Single index processed

start block = $\left| \frac{(\rho - i)}{B} \right|$ shift amount = $| (p - i) \text{ mod } B |$

operand1

Single index processed

start block = $\left| \frac{(\rho - i)}{B} \right|$ shift amount = $| (p - i) \text{ mod } B |$

operand1

1332 862 302 1402 239 819 265 1053

Multiple indexes processed

start block = $\left| \frac{(\rho - i)}{B} \right|$ shift amount = $\lfloor (p - i) \bmod B \rfloor$

operand1

1332 862 302 1402 239 819 265 1053

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Multiple indexes processed

start block = $\left| \frac{(\rho - i)}{B} \right|$ shift amount = $\lfloor (p - i) \bmod B \rfloor$

operand1

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Sampling polynomials uniformly ⁹

Generating polynomials in **R** and **R***w*

The component of the vector $h \in R$ are generated by the SHAKE-256 algorithm (a SHA-3 eXtensible Output Function) expanding the small 320-bits public seed.

The HQC specification uses the constant-time algorithm from [\[1\]](#page-56-0):

- runs in constant-time
- uses of an exact amount of randomness (32 · *w* bits)
- requires a modulo operations between a 32-bit dividend and a generic 16-bit divisor

We used a straightforward *shift-and-subtract* pipelined algorithm, not requiring DSPs to perform the operation.

At synthesis time the number of pipeline stages can be selected to balance resources usage and timing closure.

Public code: Reed Solomon Encoder

The code treats a block of data as a set of \mathbb{F}_{2^8} elements (symbols).

In a systematic encoding procedure the sequence of symbols of the message polynomial *u*(*x*) are the prefix of the codeword, and the error correcting symbols are the suffix:

$$
c(x) = x^{n_e - k_e} u(x) - (x^{n_e - k_e} u(x) \bmod g(x))
$$

A simple way to produce such special encoding is through a Linear Feedback Shift Register [\[2\]](#page-56-1):

Public code: Reed Solomon Decoder

Consider a valid codeword *c*(*x*) affected by an unknown error *e*(*x*) which has up to *t* terms:

$$
r(x) = c(x) + e(x)
$$

Decoding algorith overview

The decoder computes:

- **■** the polynomial associated to the syndrome of the received word $r(x)$
- both positions and values of the coefficients of *e*(*x*)
- the error-free codeword is derived as $c(x) = r(x) e(x)$.

Public code: Reed Solomon ¹⁰ Decoder

First, the received polynomial $r(x)$ is evaluated at each root α^i of the generator polynomial *g*(*x*) using the Horner's method, determining the *syndrome polynomial S*(*x*)

We employed the design of the Enhanced Parallel Inversionless Berlekamp-Massey Algorithm (ePIBMA) introduced in [\[3\]](#page-56-2).

The *error locator polynomial* Λ(*x*) and the *auxiliary polynomial B*(*x*) are derived from the syndrome polynomial *S*(*x*)

Public code: Reed Solomon ¹⁰

Decoder

Similarly, we used the Enhanced Chien Search and Error Evaluator design from [\[3\]](#page-56-2).

The *error evaluator polynomial* $\Omega(x)$ is computed from $\Lambda(x)$ and $B(x)$.

To derive the 128-bit codewords corresponding to each 8-bit input message, we follow the traditional message vector multiplied by the generator matrix *G*.

Working with 32-bits words, the presence of repeated words in *G* yields some identical intermediate values during the multiplication.

Consequently, the size of multiplexers and the number of XOR gates were decreased substantially.

Public code: Reed Muller 111 Decoder

The operation is carried out by a Maximum Likelihood (ML) decoder computing a fast Hadamard transform [\[4\]](#page-56-3)

We find the maximum absolute value with a pipelined comparator tree computing pairwise maxima, acting on a tunable-sized input vector.

HQC schedule ¹²

HQC specification

HQC schedule ¹²

Performance gains from 13% to 32% over the entire cryptographic primitive **without any cost or security implications**

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Experimental results ¹³

Designed in SystemVerilog, tested with CocoTB following the Universal Verification Methodology (UVM).

Synthesized on an Artix-7 xc7a200tfbg484-3 FPGA, and validated it on a Digilent's Arty A7-100T employing the (modified) official Known Answer Tests (KAT) via a UART module.

The source code is available on Zenodo:

Experimental results ¹⁴ Public code encoder/decoder

Table: Performance of the public error correction code decoder. Area-Time product in eSlices \cdot ns

Experimental results ¹⁴

Fixed weight polynomial sampler

Table: Performance of fixed-weight polynomial samplers. Area-Time product 1 in $\mathrm{eSlices}\cdot\mu\mathrm{s}$

 $1*$ Contribution of DSP units not present in the AT product

Experimental results ¹⁴ Top-level: Key Generation

Table: Performance of HQC keygen top-module w/o SHAKE256 (5520 LUTs and 2810 FFs). AT product in eSlices · ns

Experimental results 14

Top-level: Encapsulation

Table: Performance of HQC encapsulation top-modules w/o SHAKE256 (5520 LUTs and 2810 FFs). AT product in eSlices · ns

Experimental results 14

Top-level: Decapsulation

Table: Performance of HQC decapsulation top-modules w/o SHAKE256 (5520 LUTs and 2810 FFs). AT product in eSlices · ns

Conclusions ¹⁵

Our work contributes to the current state-of-the-art:

- improving both latency and efficiency of HQC Key Encapsulation Mechanism RTL designs
- detailing an efficient implementation for the public error correction code in use by HQC
- providing an optimization for the HQC algorithm significantly improving the performance of the algorithm

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